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Dynamic Calibration of Pressure Measurement Systems: An Improved Method

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Abstract

This paper describes a portable system allowing fast and accurate *in situ* calibration of a pressure measurement system. It is shown that by sacrificing unnecessary frequency resolution, the system's transfer function can be determined with better accuracy than the more commonly used white-noise method for a given sampling time.

Introduction

Measurement of fluctuating pressure at a tap connected to a remote transducer via a length of sealed tubing is a common technique in wind engineering [3]. It has also been applied to the study of airfoils [6] and in other situations where placement of a transducer at the measurement location is infeasible [2]. As pressure disturbances at the tap propagate through the tubing, they are distorted by a combination of effects resulting from "organ pipe" resonance and viscous damping. As a result, the output of the transducer does not accurately represent the time-history of pressure appearing at the tap.

Bergh and Tijdeman [1] developed a useful theoretical model for the distortion introduced by a series of connected tubes of different diameter and length, and found it to agree well with experimental observations. The basis of the model is the small-amplitude assumption which allows the tubing system to be represented as a linear, time-invariant (LTI) system. Such a system is characterized by a transfer function giving the amplitude and phase distortion of a sine wave input of a given frequency. Example transfer functions are shown in Figure 1.

Historically, the distortion was corrected by introducing restrictions into the tubing at certain locations in order to flatten the resonant peaks and produce an amplitude response as close as possible to unity, and a phase response as linear as possible. Today the correction is typically done digitally using the inverse transfer function (ITF) method first demonstrated by Irwin et al. [4]. Basically, this method amounts to convolution of the digitized transducer signal with a linear filter, the frequency response of which is the inverse of the tubing system's response. The output is thus an approximation of the pressure time-history at the tap.

The accuracy of the correction depends on the accuracy to which the tubing transfer function is known. The theoretical model can be used, however the transfer function is very sensitive to certain parameters, in particular tube internal diameter which appears in the equation raised to the 6^{th} power (and which is usually nominally given by the manufacturer and may vary slightly along the length of the tube). Figure 1 demonstrates this sensitivity. Typically if accuracy of better than ±5% is required then the system should be calibrated experimentally; in addition there are situations in which the theoretical model is not applicable (e.g. branched systems). Experimental calibration is performed by subjecting the tap to some kind of acoustic excitation; by measuring the amplitude and phase spectra of both the input and output the transfer function can be determined. Commonly the excitation signal is white noise [4], a frequency sweep of sine waves [3], a step change [5], or some other kind of periodic signal (e.g. square/triangle/sawtooth waves).

This paper describes a system which was developed to calibrate a pressure measurement system consisting of 155 taps in total, each connected to transducers by different lengths (ranging from 300–700mm) of 1mm I.D. PVC tubing. The transducers were part of a Dynamic Pressure Measurement System supplied by Turbulent Flow Instrumentation (TFI), Inc. Calibration results for three representative tubing lengths are presented, and compared to those obtained using the commonly-used white noise method.

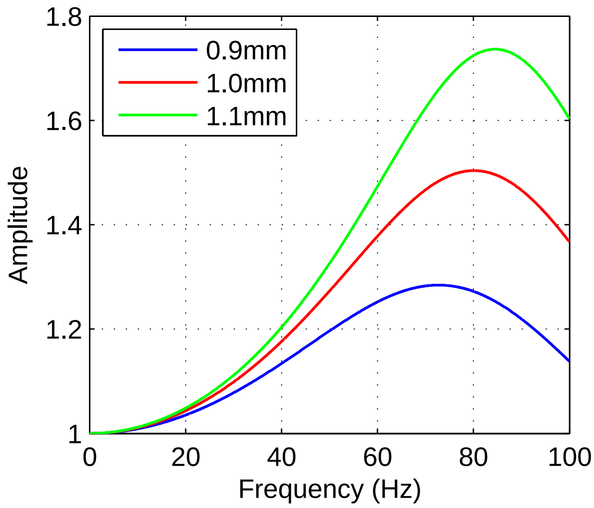

Figure 1. Amplitude response of a 750mm tube of various inner diameters.

Hardware

The device consisted of a 40mm diameter mylar cone speaker sealed to a small chamber with two ports (known as an acoustic coupler, see Figure 2). One port was 4mm in diameter and surrounded by a rubber O-ring, which could be placed over a tap being calibrated. The smaller port was connected directly to a reference transducer, also supplied by TFI, which had a flat frequency response up to 2kHz. In order to assume that the reference transducer accurately measures the pressure appearing at the tap, it is necessary that the distance between the two ports be small compared to the sonic wavelength at the frequencies of interest. Given that the frequency range of interest for the application for which the system was designed was 0–100Hz, this condition was easily met ($d/\lambda \approx 0.002$), and it is expected that the system could be used to calibrate at much higher frequencies. The rubber seal was necessary to allow the generation of low-frequency pressure fluctuations at the tap without excessive displacement of the speaker cone and distortion associated therewith.

The speaker was connected to a portable signal-generator circuit based on a PIC® microcontroller (Microchip, Inc.). The microcontroller's flash memory was pre-programmed with a wavetable representing the signal to be generated. Upon pressing a button, the microcontroller would output a pulse-width-modulated signal of approximately 4 seconds duration representing the waveform. The speaker drive signal was generated by passing the microcontroller output through a 2-pole active Chebyshev low-pass filter and then into a power amplifier.

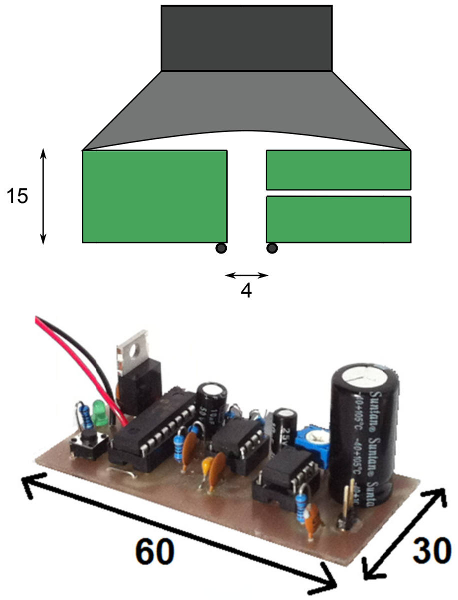

Figure 2. Dimensions of acoustic coupler (cross-section shown) and driver circuit (dimensions in mm).

Method

The system operated by subjecting a tap to a periodic signal consisting of a sum of discrete frequencies of random phase evenly spread across the frequency range of interest. In this case the range of interest was 0–100Hz, and frequencies in steps of approximately 10Hz were used. A portion of the signal and its frequency spectrum are shown in Figure 3 and Figure 4 respectively. The signal was tapered to zero at the beginning and end of the 4-second sample to avoid transients which would pollute the spectrum. In an automated post-processing computer program, the tapered portions of the sample were removed and the remaining part subjected to Fourier analysis to determine the amplitude and phase of the transfer function at each of the frequencies. By using a periodic signal, the uncertainty and spectral leakage issues inherent in spectral estimation of non-periodic signals (e.g. white noise and step response) was avoided. By spreading the power roughly equally between the constituent frequencies, the decrease in signal-to-noise ratio with increasing frequency associated with the use of square/triangle/sawtooth signals was also avoided. The short sample length necessary to obtain repeatable results (≈ 4s) made the method a lot faster than performing sine sweeps.

The disadvantage of this method (hereafter referred to as the "discrete spectrum method") is that it only provides the value of the transfer function at a discrete set of points. However, it will be shown in the following sections that the increase in accuracy with which the transfer function is determined far outweighs the error introduced by interpolating between these points.

Estimation of Interpolation Error

Since the transfer functions are expected to be continuous, and possess continuous derivatives up to an order of at least three, cubic spline interpolation performs very well. In order to estimate the error incurred by interpolation, the interpolation process was applied to the theoretical transfer functions of the three tubing lengths under investigation. Since the measured transfer functions are reasonably close to the theoretical ones (see Figure 6) this should give a reasonable order-of-magnitude estimate of the interpolation error expected in the experimental calibration.

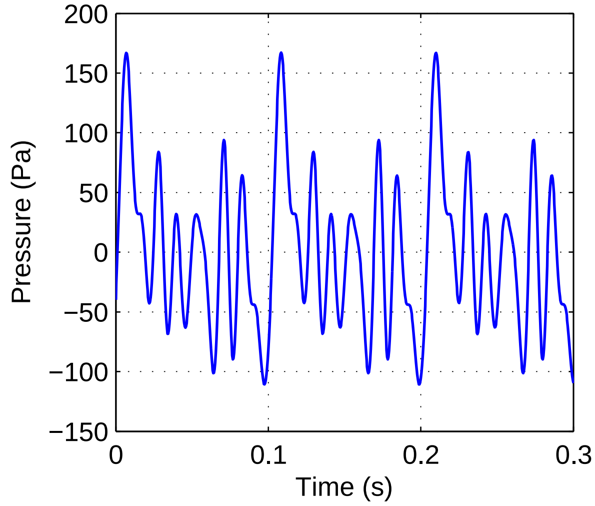

Figure 3. Part of periodic waveform used to drive speaker.

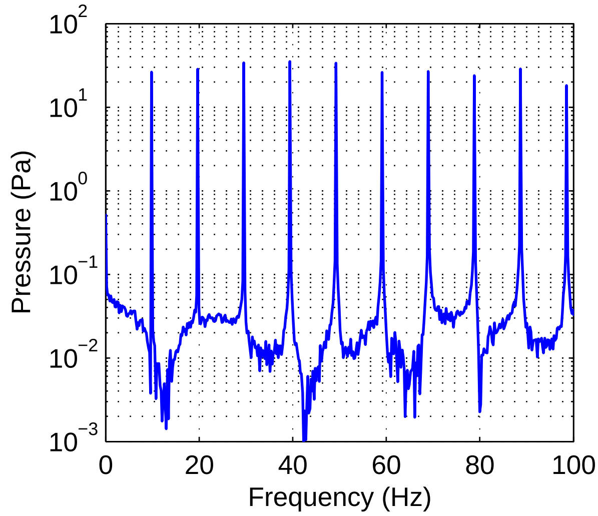

Figure 4. Spectrum of driving signal, as measured by reference transducer.

The three tubing lengths were chosen to represent cases where the first resonant peak was (a) outside the frequency range of interest, (b) at the upper limit of the range of interest, and (c) inside the range of interest. Predicted interpolation errors are shown in Figure 5.

As expected, when the upper limit of the frequency range corresponds to a region of high curvature, the interpolation loses accuracy. However the error is still small, and can be easily reduced further by including one or two extra frequencies beyond the range of interest in the calibration process (see Figure 5).

Calibration Results

The calibration process was performed 10 times on each of the three tubing systems in order to estimate the measurement variance. Results are shown in Figure 6 (note that error bars have been magnified to show $\pm 30\sigma$). These show that the amplitude response can deviate significantly from the theoretical value at frequencies surrounding the resonant peak.

In order to compare the performance of the discrete spectrum method to the white noise method, the system was temporarily reprogrammed to output a continuous white noise signal. A 4s (i.e. the same duration as used with the discrete spectrum method) portion of the signal was then used to estimate the transfer function of the 850mm tube. The resulting estimate is very noisy, as shown in Figure 7. Even when the result was averaged over 5, and then 30 realizations, significant noise remained in the amplitude estimate, although phase became reasonably smooth.

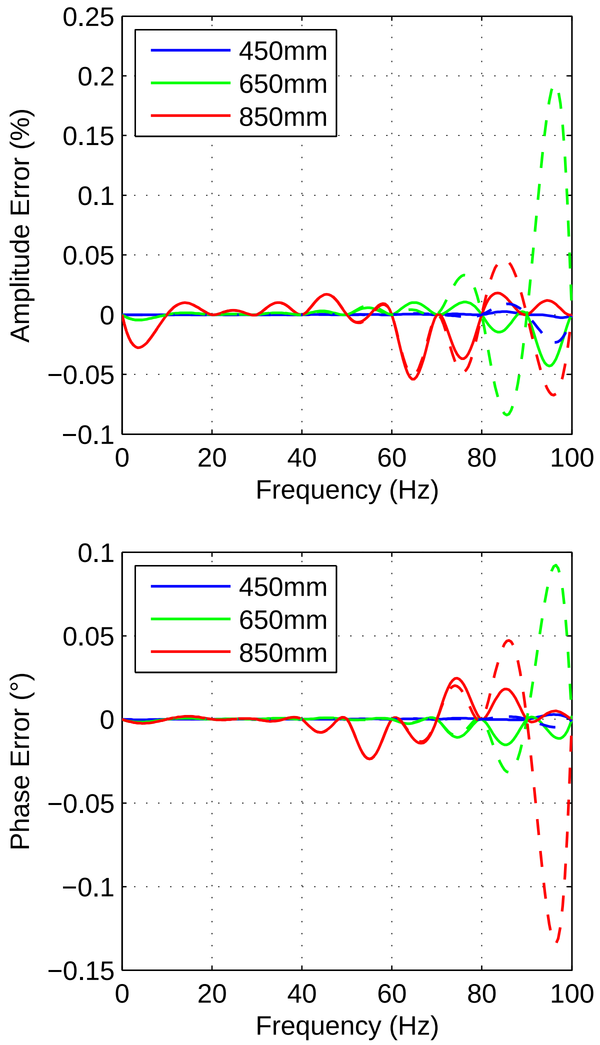

Figure 5. Amplitude and phase error resulting from cubic spline interpolation using points from 10–100Hz (– –) and from 10–120Hz (—) in steps of 10Hz.

Obviously the transfer functions obtained using the white noise method must be smoothed (typically with a moving average filter) before they can be used in applying the ITF method. Results of smoothing with a moving average filter of window length approximately 18Hz (the minimum required to produce reasonably smooth results) are shown in Figure 8. These show that even after smoothing, there is still significant variance in the resulting transfer function estimates (mainly in amplitude). The smoothing also tends to flatten the amplitude peak, the amount of flattening being dependent on the window size used. This introduces another source of error which is difficult to quantify. The measurement variance of each method at frequencies from 0–100Hz is compared in Table 1 and Table 2, showing that the discrete spectrum method far outperforms the white noise method in terms of measurement precision, even when white noise results are averaged over a longer sampling time. Note that white noise figures were obtained post-smoothing.

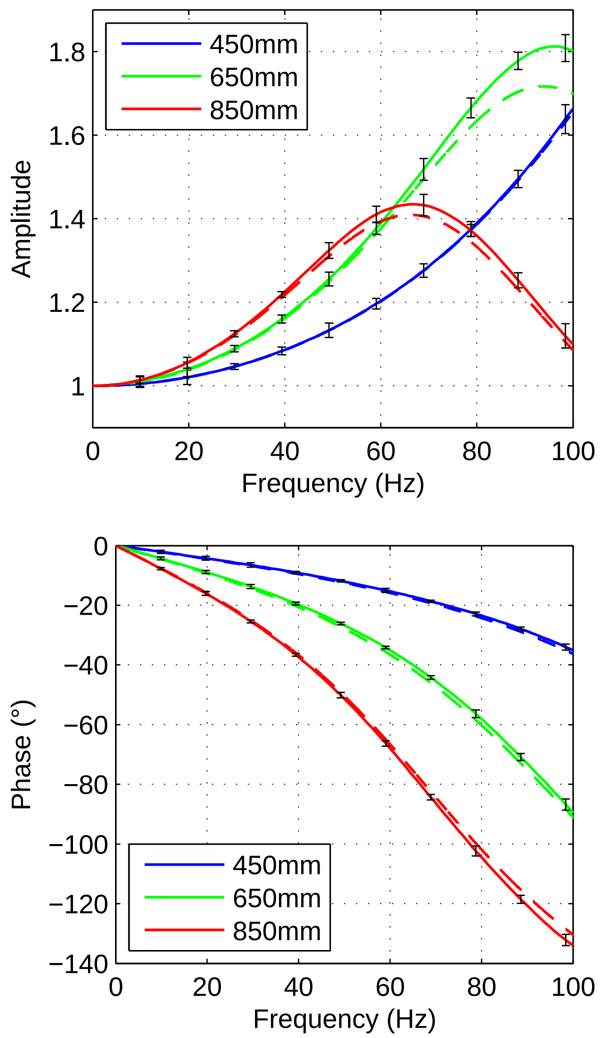

Figure 6. Amplitude and phase response as measured (—), and as calculated by the model of Bergh and Tijdeman [1] (– –). Error bars show ±30σ.

Bias Errors

The calibration process was shown to give very small precision error, but bias errors have not been addressed. These include errors in static calibration of the reference transducer, as well as non-linearity and hysteresis of all transducers. For the transducers used here these errors are estimated to be approximately 0.5%. Thus the calibration uncertainty, which usually forms a large part of the error budget in a dynamic pressure measurement, has been reduced to the point where bias error becomes the limiting factor on accuracy.

Conclusion

A portable system allowing fast and accurate *in situ* calibration of a pressure measurement system was developed. It was shown that by sacrificing unnecessary frequency resolution, the transfer function can be determined with better accuracy than the commonly used white-noise method for a given sampling time.

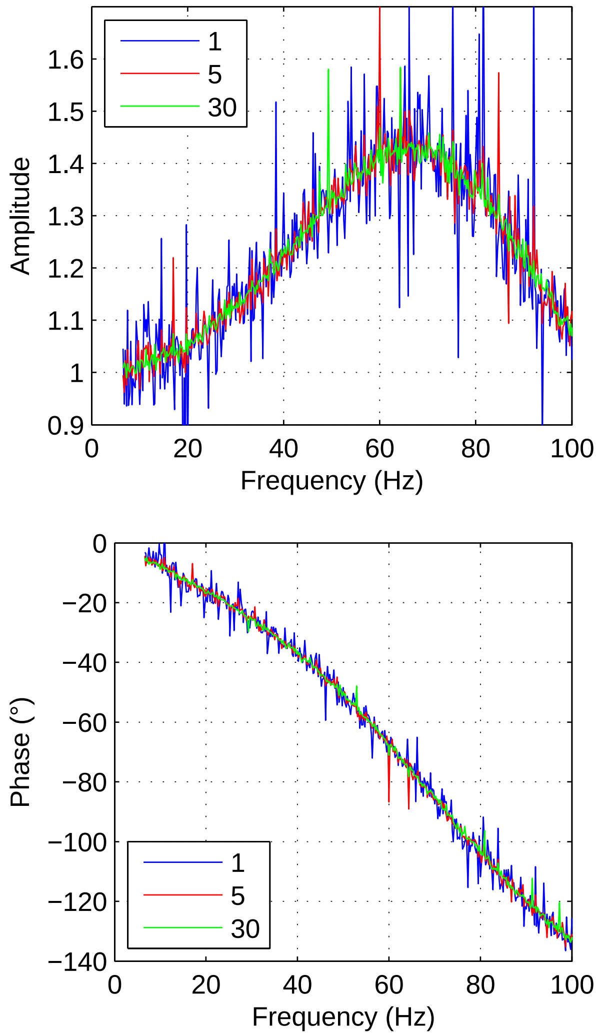

Figure 7. Amplitude and phase response of the 850mm tube obtained using the white noise method, averaging over 1, 5, and 30 realizations.

f (Hz)	$2\sigma_{amp}$ (%)		
	White noise	White noise, averaged over 5 realizations	Discrete spectrum
20	2.318	1.205	0.083
40	2.448	1.628	0.037
60	2.364	2.052	0.092
80	2.747	1.206	0.068
100	2.567	0.945	0.174

Table 1. Comparison of measurement standard deviations (amplitude) for 850mm tube.

f (Hz)	$2\sigma_{phase}$ (°)		
	White noise	White noise, averaged over 5 realizations	Discrete spectrum
20	1.231	0.687	0.042
40	1.628	0.806	0.032
60	1.775	1.312	0.058
80	2.407	1.450	0.110
100	3.527	2.516	0.124

Table 2. Comparison of measurement standard deviations (phase) for 850mm tube.

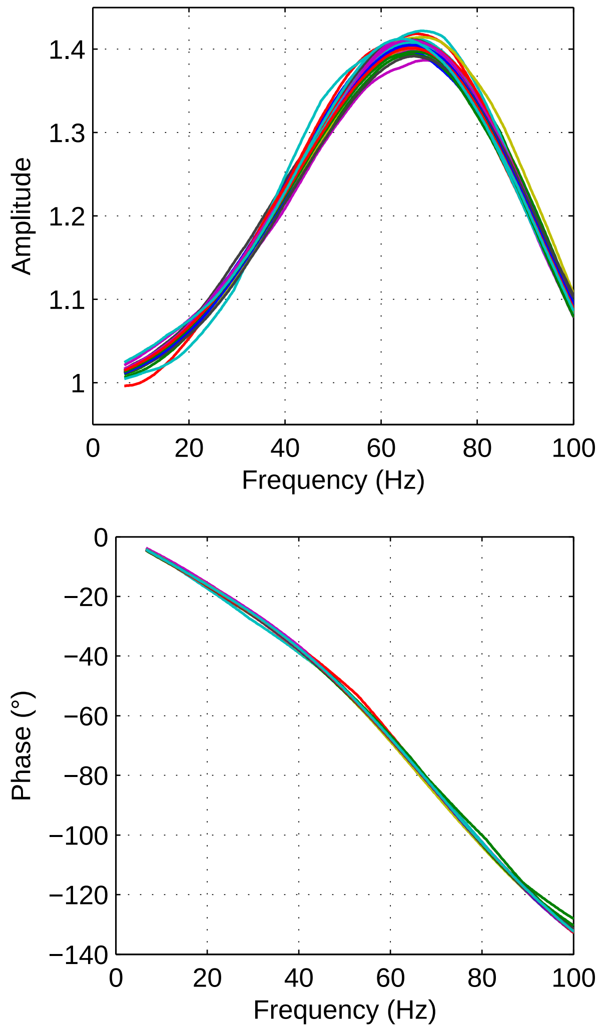

Figure 8. Amplitude and phase response obtained using the white noise method after smoothing with a moving average filter. Results of 30 repeats are plotted.

References

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